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Course of Power System Analysis

The load flow problem

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Outline

From the physical network to the admittance matrix

- Introduction
- Network nodal analysis

The equations

- Introduction
- Cartesian coordinates formulation
- Polar coordinates formulation
- Line power flows

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From the Physical Network to the Admittance Matrix - Introduction

An power system is essentially composed by:

- ***buses*** (or nodes), that can be distinguished in ***generator buses*** (corresponding to the generator terminals), ***reactive compensation buses*** (corresponding to the terminals of the synchronous compensators and the static compensators), ***interconnection buses*** (where more lines converge in order to form the «meshed» configuration) and ***load buses*** (which feed the equivalent loads seen from the High Voltage network) (see Note #1);
- ***capacitor banks*** in shunt and in series connection;
- ***transformers***;
- ***reactances***, in shunt and in series connection;
- ***lines*** overhead and cables that link the various buses;
- …

From the Physical Network to the Admittance Matrix - Introduction

NOTE #1: on the number of the network buses

A simplification of the study can be obtained by limiting the connected network at the high voltage and considering the lower voltage parts as equivalent concentrated loads.

From the Physical Network to the Admittance Matrix - Introduction

Hypothesis:

- **network in permanent state of equilibrium,**
- **network topology and parameters are constant,**
- **constant load demands,**
- **electrical components are linear,**
- **the Network is symmetrical and balanced**

In view of the above hypothesis the phase-to-ground voltages and currents can be derived at every point of the network using the direct sequence. Their frequency correspond to the electrical speed of the synchronous machines and the active and reactive powers appear constant at every given point of the network.

Therefore, the three-phase network can be studied using an equivalent one-phase network.

From the Physical Network to the Admittance Matrix - Introduction

We will deal with this study using the **relative values** → the represented voltages in such an one-phase equivalent circuit are either the phase-to-ground or the phase-to-phase ones, therefore it is useful to apply the second ones given that the power flows in the circuit are the three-phase powers <u>in per unit</u>.

Now, the load-flow problem consists in the determination of the active and reactive power values in the various elements of the passive network (lines and transformers) under permanent system conditions. - Note #2 -

From the Physical Network to the Admittance Matrix - Introduction

<u>NOTE #2:</u> on the load-flow calculation

If the network structure is known together with the admittances that compose it, by knowing the voltages in module and phase in all the network nodes, it is possible to calculate all the other electrical parameters of interest, namely the power flows and currents injected or extracted from the nodes as well as the network losses.

<u>For this reason, the 'problem of the load-flow' is practically equivalent to the determination under permanent conditions of the phasors representing the voltages in all the network nodes (state variables).</u>

From the Physical Network to the Admittance Matrix - Introduction

Why are the calculations of Load Flow performed?

- To determine **daily,** or sub-hour, **network operating conditions** that satisfy the load demand with respect to both (i) **network security** and (ii) **operating costs**. In particular, (i) the currents in the network branches should be lower than the maximum acceptable values in compliance with specific limits and (ii) the transmission losses and/or the power plants operating cost should be the minimum possible.
- To set the optimal technical and economical power system development (**planning problem**) in order to cope with progressively increasing loads over the years. The planning problem refers to both power plants and lines. It is divided as *long or medium-term planning* (5-15 years) or *short-term planning* (1-3 years);
- In the on-line (or quasi-real-time) monitoring, the load-flow calculation is used in order to be able to deploy the most appropriate **control actions**, not necessarily automatic, taking into account that measurements are acquired from various network points, typically every 2-10 seconds.

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From the Physical Network to the Admittance Matrix – The Nodal Analysis

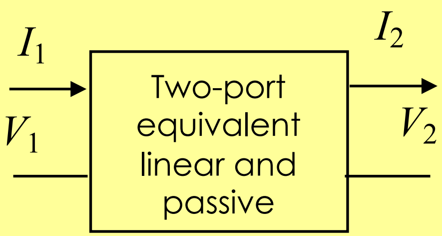
We consider a network **linear**, **passive** and **reciprocal** (we assume, for simplicity reasons, that transformers with a complex transformation ratio are not present).

A network like this is characterized by ***n+1* buses**, with ***g* generator buses** and ***u* load buses**, where the bus *n+1* is the 'neutral' (return conductor in Fig. 1). (*)

The network is characterized by ***m* branches** each one having a unique series admittance. In general, the value of such an admittance is considered, for simplicity, independent from the assumed voltage and current values (namely we neglect the inductive couplings between neighboring lines, the parameter variations - for example resistances – with the temperature and, therefore, with the ambient temperature and the current, etc.).

(*) This doesn't mean that the number of the network buses n+1 is equal to g+u+1, since, as it will be shortly clarified, it will be necessary to distinguish among the n+1 network buses an additional bus, called the 'slack bus'.

On the Two-Ports Network Equivalents



Remember that the network is supposed to be composed by branches (e.g. lines and transformers) being **passive and reciprocal** two-port network equivalents.

$$V_1 = Z_{11}I_1 - Z_{12}I_2$$
$$V_2 = Z_{21}I_1 - Z_{22}I_2$$

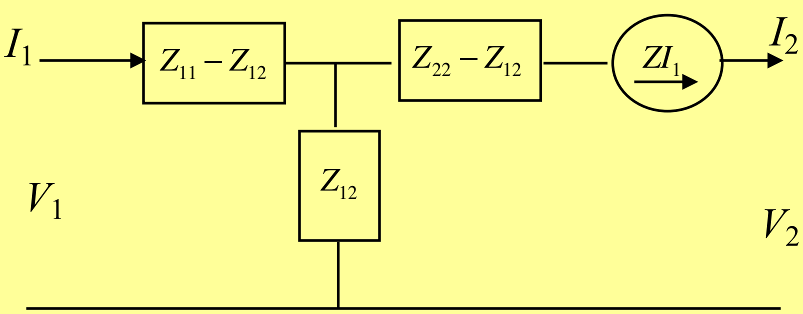
$$\mathbf{Z} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}$$

Impedance matrix in open circuit.

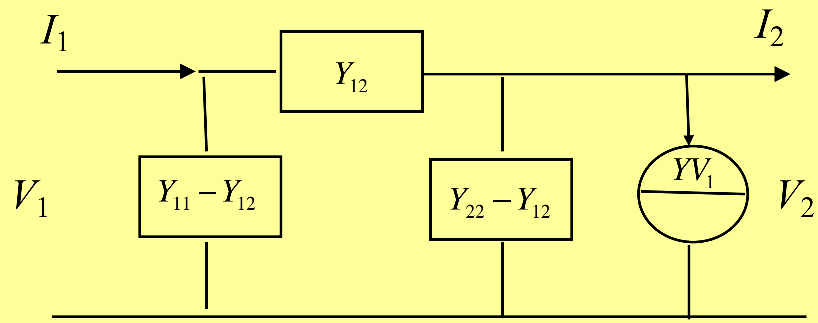
$$I_1 = Y_{11}V_1 - Y_{12}V_2$$
$$-I_2 = -Y_{21}V_1 + Y_{22}V_2$$

$$\mathbf{Y} = \begin{bmatrix} Y_{11} & -Y_{12} \\ -Y_{21} & Y_{22} \end{bmatrix}$$

Admittance matrix in short circuit.



$$Z = Z_{21} - Z_{12}$$



$$Y = Y_{12} - Y_{21}$$

On the Two-Ports Network Equivalents

$$V_1 = AV_2 + BI_2$$
$$I_1 = CV_2 + DI_2$$

$$\mathbf{T} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

$$A = \frac{Z_{11}}{Z_{21}} = \frac{Y_{22}}{Y_{21}}$$

$$B = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}} = \frac{1}{Y_{21}}$$

$$C = \frac{1}{Z_{21}} = \frac{Y_{11}Y_{22} - Y_{12}Y_{21}}{Y_{21}}$$

$$D = \frac{Z_{22}}{Z_{21}} = \frac{Y_{11}}{Y_{21}}$$

$$Z_{11} = \frac{A}{C}$$
$$Z_{12} = \frac{AD - BC}{C}$$
$$Z_{21} = \frac{1}{C}$$
$$Z_{22} = \frac{D}{C}$$

$$Z = -\frac{AD - BC - 1}{C}$$

$$Y_{11} = \frac{D}{B}$$
$$Y_{12} = \frac{AD - BC}{B}$$
$$Y_{21} = \frac{1}{B}$$
$$Y_{22} = \frac{A}{B}$$

$$Y = \frac{AD - BC - 1}{B}$$

On the Two-Ports Network Equivalents

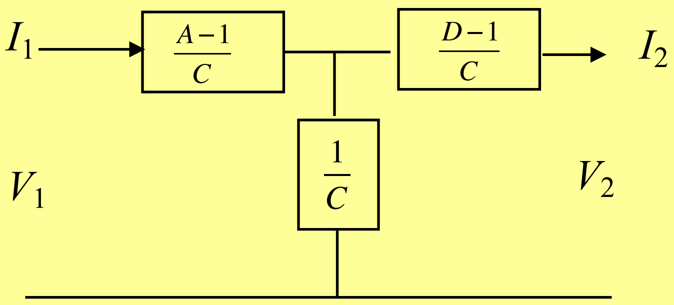
Two-port equivalent reciprocals:

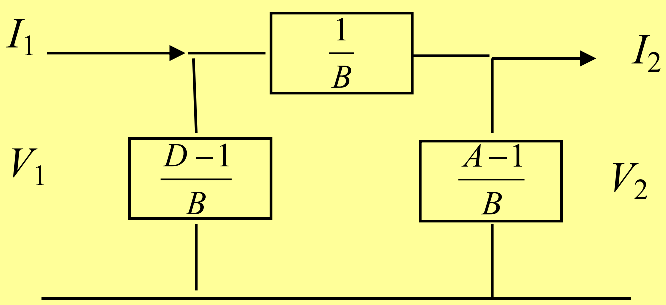
$$\left.\frac{I_1}{V_2}\right|_{V_1=0} = \left.\frac{-I_2}{V_1}\right|_{V_2=0}$$

$$AD - BC = 1$$

$$Z_{12} = Z_{21}$$

$$Y_{12} = Y_{21}$$





Symmetrical two-port network equivalents:

$$A = D$$

$$Z_{11} = Z_{22}$$

$$Y_{11} = Y_{22}$$

From the Physical Network to the Admittance Matrix – The Nodal Analysis

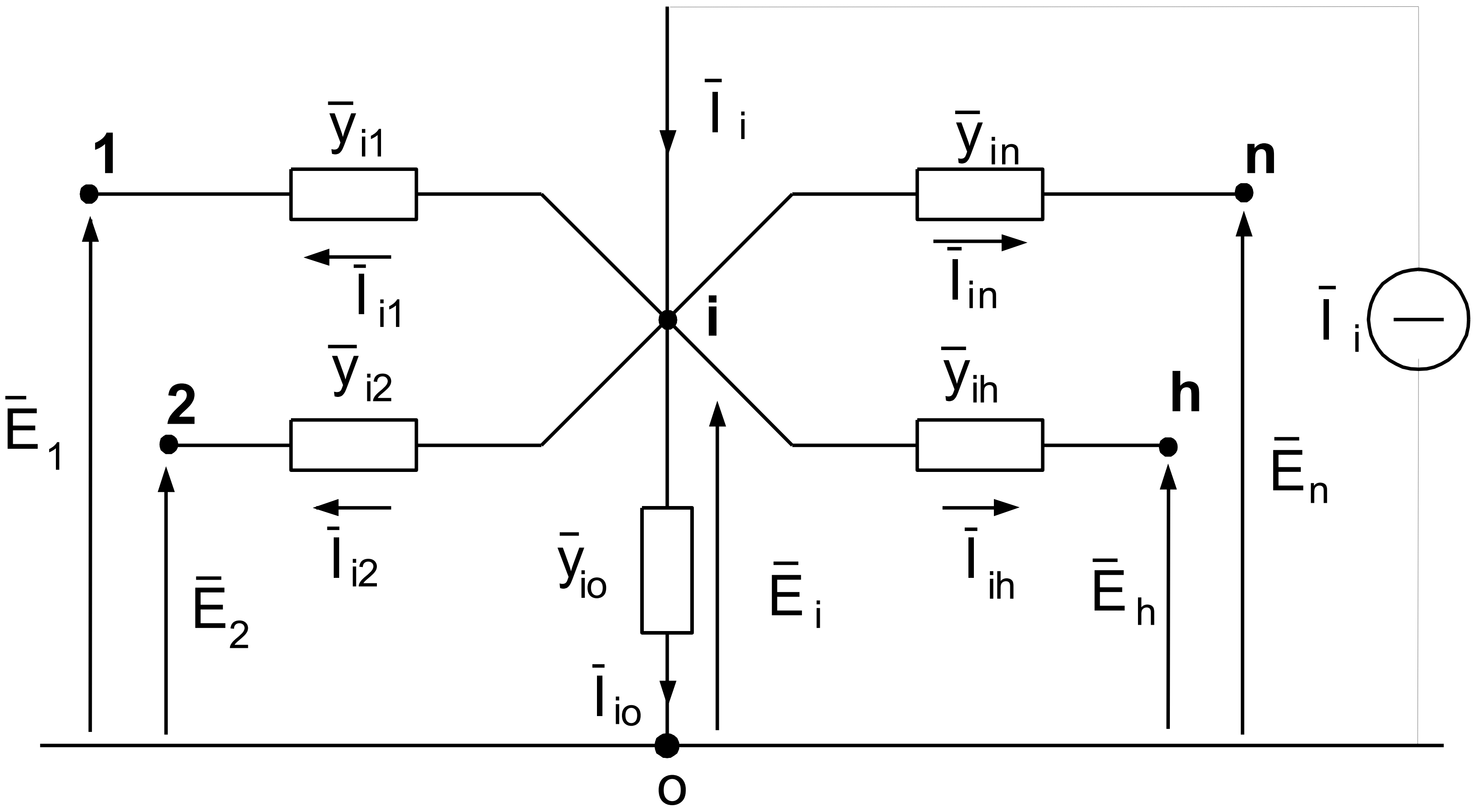


Fig. 1. Representation of the generic node i and its connections.

From the Physical Network to the Admittance Matrix – The Nodal Analysis

The problem that we would solve now (nodal analysis) is to find which are the relations between the node voltages and injected currents, considering the first as independent variables and the second as dependent variables.

For a generic network node i-th, the node voltage is indicated with $\boldsymbol{E}_\mathbf{i}$ and with $\boldsymbol{I}_\mathbf{i}$ the node-injected current (this last is the current delivered by a generator or absorbed by a load connected to the node). By convention, **a current that is injected by a generator** (into network) **is considered with positive sign and a current absorbed by a load is considered with negative sign**. If a node has only the interconnection function (i.e. it does not have generators or loads connected to it), the corresponding injected current is, obviously, null. If more generators and loads are connected to a node, the node current is the algebraic sum of the corresponding complex currents. We indicate with $\boldsymbol{I}_\mathbf{ij}$ the current of the branch that connects the nodes i and j. The complex admittance between node i and node j is indicated with $\boldsymbol{y}_\mathbf{ij}$, whereas, with $\boldsymbol{y}_\mathbf{i0}$ we indicate the sum of the admittances existing between the node i and the neutral.

From the Physical Network to the Admittance Matrix – The Nodal Analysis

With reference to the network represented in Fig. 1, the currents exiting from the node are:

$$\begin{cases} \bar{I}_{io} = \bar{y}_{io}\bar{E}_i \\ \bar{I}_{i1} = \bar{y}_{i1}(\bar{E}_i - \bar{E}_1) \\ \cdots\cdots\cdots\cdots \\ \bar{\mathrm{I}}_{in} = \bar{y}_{in}(\bar{E}_i - \bar{E}_n) \end{cases}$$

Applying the first Kirchhoff law, we obtain:

$$\bar{I}_i = \bar{y}_{io}\bar{E}_i + \bar{y}_{i1}(\bar{E}_i - \bar{E}_1) + \ldots + \bar{y}_{in}(\bar{E}_i - \bar{E}_n) =$$

$$= (\bar{y}_{io} + \bar{y}_{i1} + \ldots + \bar{y}_{in})\bar{E}_i - \bar{y}_{i1}\bar{E}_1 - \ldots - \bar{y}_{in}\bar{E}_n =$$

$$= (\bar{y}_{io} + \bar{y}_{i1} + \ldots + \bar{y}_{in})\bar{E}_i - \sum_{\substack{h=1 \\ h\neq i}}^{n} \bar{y}_{ih}\bar{E}_h$$

From the Physical Network to the Admittance Matrix – The Nodal Analysis

By setting

$$\overline{Y}_{ii} = \overline{y}_{io} + \overline{y}_{i1} + \ldots + \overline{y}_{in} = \sum_{h=0}^{n} \overline{y}_{ih}$$

$$\overline{Y}_{i1} = -\overline{y}_{i1}$$

$$\cdots\cdots$$

$$\overline{Y}_{in} = -\overline{y}_{in}$$

we obtain

$$\overline{I}_{i} = \overline{Y}_{i1}\overline{E}_{1} + \overline{Y}_{i2}\overline{E}_{2} + \ldots + \overline{Y}_{ii}\overline{E}_{i} + \ldots + \overline{Y}_{in}\overline{E}_{n} = \sum_{h=1}^{n} \overline{Y}_{ih}\overline{E}_{h}$$

From the Physical Network to the Admittance Matrix – The Nodal Analysis

A similar equation can be written for all other nodes →

for the whole network we can derive the following system:

$$
\begin{cases}
\bar{I}_1 = \bar{Y}_{11}\bar{E}_1 + \ldots + \bar{Y}_{1h}\bar{E}_h + \ldots + \bar{Y}_{1n}\bar{E}_n \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\bar{I}_h = \bar{Y}_{h1}\bar{E}_1 + \ldots + \bar{Y}_{hh}\bar{E}_h + \ldots + \bar{Y}_{hn}\bar{E}_n \\
\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\
\bar{I}_n = \bar{Y}_{n1}\bar{E}_1 + \ldots + \bar{Y}_{nh}\bar{E}_h + \ldots + \bar{Y}_{nn}\bar{E}_n
\end{cases}
$$

in matrix formulation

$$
\begin{bmatrix} \bar{I}_1 \\ \ldots \\ \bar{I}_h \\ \ldots \\ \bar{I}_n \end{bmatrix}
=
\begin{bmatrix}
\bar{Y}_{11} & \ldots & \bar{Y}_{1h} & \ldots & \bar{Y}_{1n} \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
\bar{Y}_{h1} & \ldots & \bar{Y}_{hh} & \ldots & \bar{Y}_{hh} \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
\bar{Y}_{n1} & \ldots & \bar{Y}_{nh} & \ldots & \bar{Y}_{nn}
\end{bmatrix}
\cdot
\begin{bmatrix} \bar{E}_1 \\ \ldots \\ \bar{E}_h \\ \ldots \\ \bar{E}_n \end{bmatrix}
\Rightarrow \left[\bar{I}\right] = \left[\bar{Y}\right]\left[\bar{E}\right]
$$

$\left[\bar{Y}\right]$ is the so-called network **admittance matrix**

From the Physical Network to the Admittance Matrix – The Nodal Analysis

Properties of the nodal admittance matrix elements.

- a generic element $\bar{Y}_{ij}$ out of the main diagonal, called trans-admittance, is equal to the opposite of the admittance $\bar{y}_{ij}$ of the branch that connects the nodes i and j:

$$\bar{Y}_{ij} = -\bar{y}_{ij} = \bar{I}_i \Big|_{\substack{\bar{E}_j=1 \\ \bar{E}_h=0 \quad \forall h\neq j}}$$

- a generic element $\bar{Y}_{ii}$ of the main diagonal, called self-admittance, is equal to the sum of all the admittances of the branches that are connected to node i including the ones with the neutral:

$$\bar{Y}_{ii} = \bar{y}_{io} + \sum \bar{y}_{ij} = \bar{I}_i \Big|_{\substack{\bar{E}_i=1 \\ \bar{E}_j=0 \quad \forall j\neq i}}$$

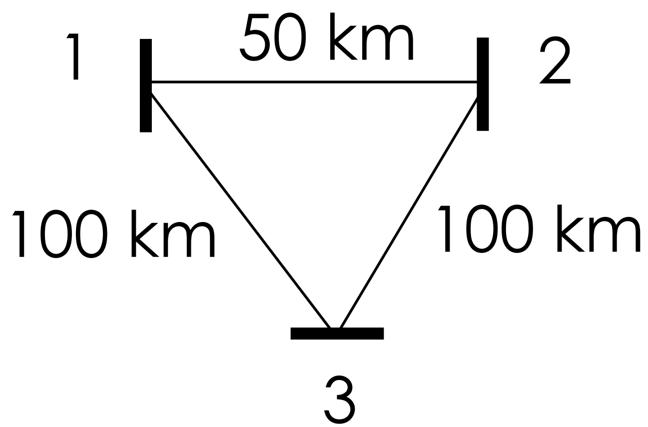
where the summation is extended to all of the nodes connected to node i.

From the Physical Network to the Admittance Matrix – The Nodal Analysis

Matrix $\left[\bar{Y}\right]$ is sparse. Furthermore, it is **diagonal-dominant** because each of its diagonal elements, in absolute value, is not lower than the sum of the other elements in the same row.

Matrix $\left[\bar{Y}\right]$ is also **symmetric** if all the double bipolars that compose the network are **reciprocal**.

Example



Lines rated voltage: 220kV

$r = 0{,}0717\ \Omega/\mathrm{km}$
$x = 0{,}424\ \Omega/\mathrm{km}$
$b = 2{,}64\ \mu\mathrm{S/km}$
$g = 0$

We assume **as power base 100 MW and base voltage 220 kV**

rpu=r*100/220^2
xpu=x*100/220^2
bpu=b*220^2/100

y12=1/((rpu+i*xpu)*50)
y13=1/((rpu+i*xpu)*100)
y23=1/((rpu+i*xpu)*100)
y10=1/2*(bpu*50+bpu*100)
y20=1/2*(bpu*50+bpu*100)
y30=1/2*(bpu*100+bpu*100)

Y11=y12+y13+y10 5,63 – j 33,2	Y12=-y12 -3,75 + j 22,2	Y13=-y13 -1,88 + j 11,1
	Y22=y12+y23+y20 5,63 – j 33,2	Y23=-y23 -1,88 + j 11,1
		Y33=y13+y23+y30 3,75 – j 22,1

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The Equations - Introduction

$$\left[\bar{I}\right]=\left[\bar{Y}\right]\left[\bar{E}\right]$$

In a network with n nodes, the n complex voltages and the n complex node currents are linked by n equations with complex variables and coefficients representing the **internal network constraints**.

The $\mathbf{2n}$ **complex voltages** and **currents** are equivalent to $\mathbf{4n}$ **real variables**. On the other hand, we have $\mathbf{2n}$ **linear equations** with real variables and coefficients, which are obtained by separating the real and imaginary parts (or the modules and the arguments).

Therefore, from the $4n$ real variables, $2n$ can be fixed arbitrarily and the remaining $2n$ are calculated by solving the system of equations of the network.

When the system is solved, and therefore all voltages and currents are known, we can calculate (see Note #2):

- P and Q inserted or extracted from the nodes
- branches powers/currents
- network losses (both active and reactive, corresponding to the balance between powers inserted and extracted from the nodes).

If the network operation conditions could be represented imposing as external constraints only voltage and current amplitudes and phases, the power flows can be calculated by solving a simple system of linear equations.

The Equations - Introduction

In practice, the operating conditions imposed on the networks (***external constraints***) are expressed by **fixing other parameters**. This implies, as will be explained shortly, that the system of equations to be solved becomes non-linear. In particular:

In the **load buses** P and Q demands are normally fixed ($\boldsymbol{P_i}$* and $\boldsymbol{Q_i}$*).

In practice, it is not appropriate to represent the various user devices with constant admittances (i.e., asynchronous motors absorb active power almost independently of the voltage, with variations in the range of +10%; gas-discharge lamps and incandescent lamps, even if they absorb power that varies with the voltage, do not follow the quadratic law).

$$P = P_0 \left(\frac{V}{V_0} \right)^{\alpha_p}$$

The dependence on the voltage is expressed by the general relations shown at the right, where the value of the exponential coefficients depend to the nature of the load and, in some cases, it can also be set = 0.

$$Q = Q_0 \left(\frac{V}{V_0} \right)^{\alpha_q}$$

The Equations - Introduction

For the **generator buses**, it is convenient to fix the P that is injected from them to the grid ($\boldsymbol{P_i}$*) and the amplitude of voltage E ($\boldsymbol{E_i}$*).

- We choose the value equal to the P that each power plant is called to provide in accordance with the plan of the global network load distribution among the production installations.
- Fixing the E, rather than the Q, is convenient for the following reasons:
  1. fixing the voltage (typically at a value between E_n and $1.1E_n$ according to the location of the power plants and the distance with respect to the loads), means that we set the voltage in the network key points (often scattered throughout the network). So the solution of the equations provides a solution acceptable for the network operation. It is also facilitated the convergence of the iterative procedure for the solution of the equations (which is not discussed here).
  2. The Q of the generators can vary between the Max-limit (i.e., generators over excitation) and the Min-limit (i.e., generators under excitation) by simply by varying the excitation current. Therefore, it is convenient to accept to operate each power plant with the Q that is provided by the calculation and which allows to obtain the predetermined voltages.

The Equations - Introduction

We have justified that both for loads and generators, it is convenient to fix the P.

It should be noted, however, that it is not possible to assign arbitrary valuesof P at all nodes because this would be equal to arbitrarily setting the network losses, which is clearly absurd. In fact, the losses are not known initially, but are calculated together with the power flows, after having solved the equations.

It is therefore allowable to arbitrarily set no more than $(n\text{-}1)$ active powers.

Consequently, for one of the nodes, that can be chosen to coincide in the numbering with the n-th node, the amplitude and the phase of the voltage are fixed. This node is called **slack bus**, as the active power, for this node, is equal to the balance between the active powers of generators/loads and the power losses.

As slack bus we can choose a generator where a significant power is installed. In this node the phase of the voltage is fixed to zero; this is equivalent to measuring the phases of the other $(n\text{-}1)$ node voltages using as a **reference the slack bus voltage phasor**.

The Equations - Introduction

Summary of the parameters that are imposed and the ones that need to be determined for the various types of buses

Type of bus	Imposed parameters (in total $2n$)		Parameters to be determined (in total $2n$)	
Generator buses	P_g	E_g	Q_g	$\arg(E_g)$
Load buses	P_c	Q_c	E_c	$\arg(E_c)$
Slack bus	E_n	$\arg(E_n) = 0$	P_n	Q_n

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The Equations - Cartesian Coordinates Formulation

We will use the following notations:

$\bar{E}_i = E'_i + jE''_i$ voltage of the i-th node;

$\bar{E}_h = E'_h + jE''_h$ voltage of the h-th node;

$\bar{Y}_{ih} = G_{ih} + jB_{ih}$ ih element of the admittance matrix $\left[\bar{Y}\right]$;

The complex power injected, or absorbed, from the i-th node can be written as:

$$\bar{S}_i = P_i + jQ_i = \bar{E}_i \underline{I}_i$$

replacing the expression that gives the complex current inserted or extracted from the node i we get:

$$\bar{S}_i = \bar{E}_i \sum_{h=1}^{n} \underline{Y}_{ih} \underline{E}_h = \left(E'_i + jE''_i\right) \sum_{h=1}^{n} \left(G_{ih} - jB_{ih}\right)\left(E'_h + jE''_h\right)$$

The Equations - Cartesian Coordinates Formulation

Then, the injected active and reactive powers of the *i*-th node will be:

$$\begin{cases} P_i = E'_i \sum_{h=1}^{n} \left(G_{ih} E'_h - B_{ih} E''_h \right) + E''_i \sum_{h=1}^{n} \left(B_{ih} E'_h + G_{ih} E''_h \right) \\ Q_i = -E'_i \sum_{h=1}^{n} \left(B_{ih} E'_h + G_{ih} E''_h \right) + E''_i \sum_{h=1}^{n} \left(G_{ih} E'_h - B_{ih} E''_h \right) \end{cases}$$

The module (the square) of the voltage at the i-th node will also be:

$$E_i^2 = E_i^{'2} + E_i^{''2}$$

The Equations - Cartesian Coordinates Formulation

The entire system of equations in cartesian coordinates assumes the following form:

$$0 = E_i''$$

$i=n$ for the unique slack bus

$$E_i^{*2} = E_i'^2 + E_i''^2$$

$i=1,2,\dots,g$ and $i=n$, for the g generator buses + the slack bus

$$P_i^* = E_i' \sum_{h=1}^{n} \left(G_{ih} E_h' - B_{ih} E_h'' \right) + E_i'' \sum_{h=1}^{n} \left(B_{ih} E_h' + G_{ih} E_h'' \right)$$

$i=1,2,\dots,g+u$, for the g generator buses + u load buses

$$Q_i^* = -E_i' \sum_{h=1}^{n} \left(B_{ih} E_h' + G_{ih} E_h'' \right) + E_i'' \sum_{h=1}^{n} \left(G_{ih} E_h' - B_{ih} E_h'' \right)$$

$i=g+1,\dots, g+u$ for the load buses

The number of equations is: $1+(g+1)+(g+u)+u=2(g+u+1)=2n.$

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The Equations – Polar Coordinates Formulation

By indicating with φ_i, θ_i respectively the arguments of the current and voltage of node i and with γ_{ih} the argument of the admittance we can write $\bar{Y}_{ih}$:

$$\bar{E}_i = E_i e^{j\theta_i}$$ voltage at the *i*-th node;

$$\bar{I}_i = I_i e^{j\phi_i}$$ current at the *i*-th node;

$$\bar{Y}_{ih} = Y_{ih} e^{j\gamma_{ih}}$$ element *ih* of the admittance matrix $[\bar{Y}]$;

The complex power at the i-th node can be written as:

$$\bar{S}_i = P_i + jQ_i = \bar{E}_i \underline{I}_i$$

Using again the network admittance matrix to express the injected node current, we obtain:

$$\bar{S}_i = \bar{E}_i \sum_{h=1}^{n} \underline{Y}_{ih} \underline{E}_h = \sum_{h=1}^{n} \bar{E}_i \underline{Y}_{ih} \underline{E}_h = \sum_{h=1}^{n} E_i E_h Y_{ih} e^{j(\theta_i - \theta_h - \gamma_{ih})}$$

The Equations – Polar Coordinates Formulation

Then, the active and reactive powers at the i-th node will be:

$$\begin{cases} P_i = \sum_{h=1}^{n} E_i E_h Y_{ih} \cos\left(\theta_i - \theta_h - \gamma_{ih}\right) \\ Q_i = \sum_{h=1}^{n} E_i E_h Y_{ih} \sin\left(\theta_i - \theta_h - \gamma_{ih}\right) \end{cases} \qquad \text{(LF.18)}$$

The Equations – Polar Coordinates Formulation

The system of equations for the solution of the load flow problem in <u>polar coordinates</u> assumes therefore the following form:

$$0 = \theta_i$$

i=n for the unique slack bus

$$E_i^* = E_i$$

i=1,2,...,g and i=n, for the g generator buses + the slack bus

$$P_i^* = E_i \sum_{h=1}^{n} E_h Y_{ih} \cos\left(\theta_{ih} - \gamma_{ih}\right)$$

i=1,2,...,g+u, for the g generator buses + the u load buses

$$Q_i^* = E_i \sum_{h=1}^{n} E_h Y_{ih} \sin\left(\theta_{ih} - \gamma_{ih}\right)$$

i=g+1,..., g+u, for the load buses

The number of equations is: $1+(g+1)+(g+u)+u=2n$.

The Equations – Polar Coordinates Formulation

In this case, the first $g+2$ equations (generator buses and slack bus) are the positions, which can be replaced directly in the other equations of the system, so the number of equations needed in the formulation in polar coordinates ($g+2u$) is lower than the one in Cartesian coordinates. This is due to the fact that the modules of the voltages at the generator and the phase of the voltage of the slack bus, are already acquired as imposed parameters.

This does not necessarily imply a reduction of the computation time. In fact, using polar coordinates, it is necessary to calculate trigonometric functions sin and cos.

The Equations – Polar Coordinates Formulation

Formulation in polar coordinates for the voltage and cartesian for the admittances:

$$0 = \theta_n''$$

i=n for the unique slack bus

$$E_i^* = E_i$$

i=1,2,...,g and i=n, for the g generator buses + the slack bus

$$P_i^* = E_i \sum_{h=1}^{n} E_h \left(G_{ih} \cos\theta_{ih} + B_{ih} \sin\theta_{ih} \right)$$

i=1,2,...,g+u, for the g generator buses + the u load buses

$$Q_i^* = E_i \sum_{h=1}^{n} E_h \left(G_{ih} \sin\theta_{ih} - B_{ih} \cos\theta_{ih} \right)$$

i=g+1,..., g+u, for the load buses

Outline

From the physical network to the admittance matrix

- Introduction
- Network nodal analysis

The equations

- Introduction
- Cartesian coordinates formulation
- Polar coordinates formulation
- Line power flows

The Equations – Line Power Flows

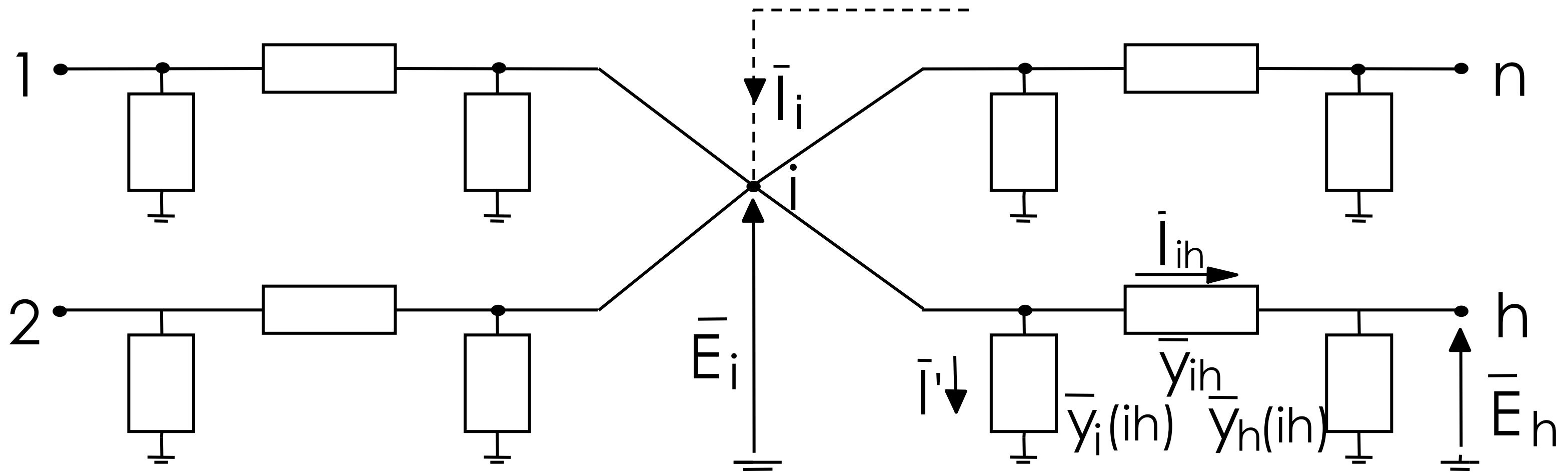

Fig.LF_2. Power flow in the line *ih*.

$$\bar{S}_{ih} = P_{ih} + jQ_{ih} = \bar{E}_i\left(\underline{E}_i - \underline{E}_h\right)\underline{y}_{ih} + E_i^2\,\underline{y}_{i(ih)} \qquad \text{(LF.20)}$$

Polar

$$\bar{S}_{ih} = \bar{E}_i\left(\underline{E}_i - \underline{E}_h\right)\underline{y}_{ih} + E_i^2\,\underline{y}_{i(ih)}$$

$$= E_i e^{j\theta_i}\left(E_i e^{-j\theta_i} - E_h e^{-j\theta_h}\right) y_{ih} e^{-j\gamma_{ih}} + E_i^2 y_{i(ih)} e^{-j\gamma_i} \qquad \text{(LF.21)}$$

$$= E_i^2 y_{ih} e^{-j\gamma_{ih}} - E_i E_h y_{ih} e^{j(\theta_i - \theta_h - \gamma_{ih})} + E_i^2 y_{i(ih)} e^{-j\gamma_i}$$

Having defined with γ_i the argument of the admittance $y_i(ih)$

The Equations – Line Power Flows

Formulation in cartesian coordinates:

$$P_{ih} = \left(g_{ih} + g_{i(ih)}\right)\left(E_i'^2 + E_h''^2\right) - g_{ih}\left(E_i'E_h' + E_i''E_h''\right) + b_{ih}\left(E_i'E_h'' - E_h'E_i''\right)$$

$$Q_{ih} = -\left(b_{ih} + b_{i(ih)}\right)\left(E_i'^2 + E_h''^2\right) + g_{ih}\left(E_i'E_h'' - E_h'E_i''\right) + b_{ih}\left(E_i'E_h' + E_i''E_h''\right)$$

Formulation in polar coordinates for the voltage and cartesian for the admittances:

$$P_{ih} = E_i^2\left(g_{ih} + g_{i(ih)}\right) - E_iE_h\left(g_{ih}\cos\theta_{ih} + b_{ih}\sin\theta_{ih}\right)$$

$$Q_{ih} = -E_i^2\left(b_{ih} + b_{i(ih)}\right) - E_iE_h\left(g_{ih}\sin\theta_{ih} - b_{ih}\cos\theta_{ih}\right)$$

The numerical solution

Since the equations that link the unknown network parameters with those that are known are non-linear, they must be resolved by using iterative numerical procedures (e.g., Newton-Raphson, Gauss-Seidel methods) starting from a **reasonable initial profile** (**for instance: all the unknown amplitudes set equal to 1 per unit or to the value of the slack bus, all the unknown phases set equal to the phase of the slack bus**), they are progressively updated until the convergence, according to one of the procedures provided by the numerical analysis. The selection of the initial profiles is generally such that, if the process converges, it can guarantee that **<u>the convergence to one of the solutions has a physical meaning</u>**. The most common iterative methods are based on the description of the network in terms of the nodal admittance matrix, although there are also different procedures.

The numerical solution of the load-flow problem is beyond the scope of this course and we leave the students to use dedicated software packages allowing for the solution of this fundamental power-systems problem.